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A survey on Magic Labeling of digraphs

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Abstract: *In this paper we study the different type of magic labelings for digraphs.Graph labeling is widely used in coding theory, designing of electrical circuits, radar, astronomy, communication networks etc.*
A magic labeling on a graph G with v vertices and e edges is defined as a one-to-one map taking the vertices and edges onto the integers 1, 2, ... , v+e with the property that the sum of the label on an edge and the labels of its endpoints is constant independent of the choice of edge.(3).A few graphs for which magic labelling has not yet been assigned is discussed.
Keywords - *vertex set of digraph D is V(D), edge set of digraph D is E(D).*

I. Introduction

A graph is a finite non- empty set of objects called vertices together with a set of unordered pairs of distinct vertices of G called edges. The vertex set of G is denoted by V(G), while the edge set is denoted by E(G).

A directed graph or digraph D is a finite non- empty set of objects called vertices together with a set of ordered pairs of distinct vertices of D called arcs. As with graphs, the vertex set of D is denoted by V(D) and the arc set is denoted by E(D).

A graph labelng is an assignment of real values or subsets of a set to the vertices or edges or both subject to certain conditions. Clearly, in the absence of additional constraints, every graph can be labeld in infinitely many ways. Thus, utilisation of labeled graph models requires imposition of additional constraints which characterize the problem being investigated. A wide variety of these labelings has been studied such as, graceful labeling, sequential labeling, harmonious labeling and set labeling etc.

Labeled graphs are becoming an increasingly useful family of mathematical models for a broad range of applications. The usage of these labeled graphs are found in various coding theory problems, including the design of good radar-type codes. They have also been applied to determine ambiguities in X-ray crystallographic analysis, to design a communication network addressing system, to determine optimal circuit layouts, and to problems in analytic number theory.(1)

II. Literature Review

So far, a lot of research work has been done on both vertex and edge labeling of undirected graphs. Further objective is to study analogous definitions for labelings of directed graphs.The different types of labelings imposed on digraphs are as follows and all these labeling are referred from (2)

I.Magic Labeling:

A magic labeling of a digraph D is a one-to-one map λ from $V(D) \cup E(D)$ onto the set of integers $\{1,2, \ldots \ldots, v+e\}$ in which all the sums

$$m_A(x) = \lambda(x) + \sum_{x \to y} \lambda(x,y)$$

and all the sums

$$m_B(x) = \lambda(x) + \sum_{z \to x} \lambda(z,x)$$

are constant, independent of the choice of x.A digraph with a magic labeling is a magic digraph.(2)

For Example:

3 7

4 6

1

In the above example we observe that the digraph is magic with magic sum 12.
Kotzig and Rosa defined a magic labeling to be a total labeling in which the labels are the integers from 1 to $|V(G)| + |E(G)|$ and the sum of labels on an edge and its two endpoints are constant; this constant is called the magic sum of the labeling. In 1996 Ringel and Lladoredefined this type of labeling and called the labelings edge-magic.Total labelings have also been studied in which the sum of the labels of all edges adjacent to the vertex x ,plus the label of x itself,is constant.

II. Locally Magic Labelings

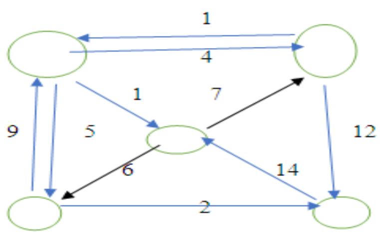
A locally magic labeling or magic vertex labeling on a digraph D is a one-to-one map λ from E(D) into a subset of integers $\{1,2, \ldots . v + e\}$,where $v = |V(D)|$ and $e = |E(D)|$ in which the sum

$$m_A(x) = \sum_{x \to y} \lambda(x, y)$$

and the sum

$$m_B(x) = \sum_{z \to x} \lambda(z, x)$$

Are constant for each x,but this constant need not be the same for each vertex x in D(A) digraph with a locally magic labeling is called a locally magic digraph .A strongly locally magic labeling in which only the labels $\{1,2, \ldots . e\}$ are used then such type of digraphs is called strongly locally magic Example: Given below is an example of Locally Magic digraph



III. Vertex-Magic Edge labelings

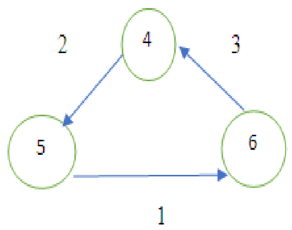
A digraph D with v vertices and e edges has a vertex-magic edge labeling if there is a bijection $\lambda: E(D) \to \{1,2, \ldots e\}$ such that for every x

$$\sum_{(y,x) \in E(D)} \lambda(y, x) = \sum_{(x,y) \in E(D)} \lambda(x, y) = k$$

For some $k \in N$.If such a labeling exists, the digraph is said to be vertex-magic.
Theorem: If a digraph D with v vertices and e edges is vertex-magic, then $2v/(e + 1)e$.(2)

IV. Arc-Magic Labelings

An arc-magic labeling on a digraph D is a bijective map λ from V(D)UE(D) onto the integers $\{1,2, \ldots . v + e\}$,where $v = |V(D)|$ and $e = |E(D)|$ in which the sum $\lambda(xy) + \lambda(y)$ is constant for every arc xy in D.
Example:



The above digraph is arc-magic with magic constant 7
Theorem: If a digraph has arc-magic labeling, then each vertex of that digraph has in-degree at most one

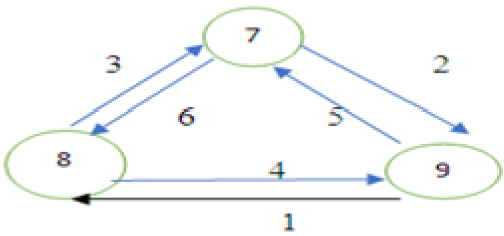
V. In/Out Magic Total Labelings
A digraph D with v vertices and e edges is in-magic if there exists a bijective function
$\lambda: V(D) \cup E(D) \to \{1,2, \dots, v+e\}$ such that for every x

$$\lambda(x) + \sum_{(x,y)\in E(D)} \lambda(y,x) = k$$

For some $k \in Z$
Similarly, a digraph D with v vertices and e edges is out-magic if there exists a bijective function $\lambda: V(D) \cup E(D) \to \{1,2, \dots, v+e\}$ such that for every x

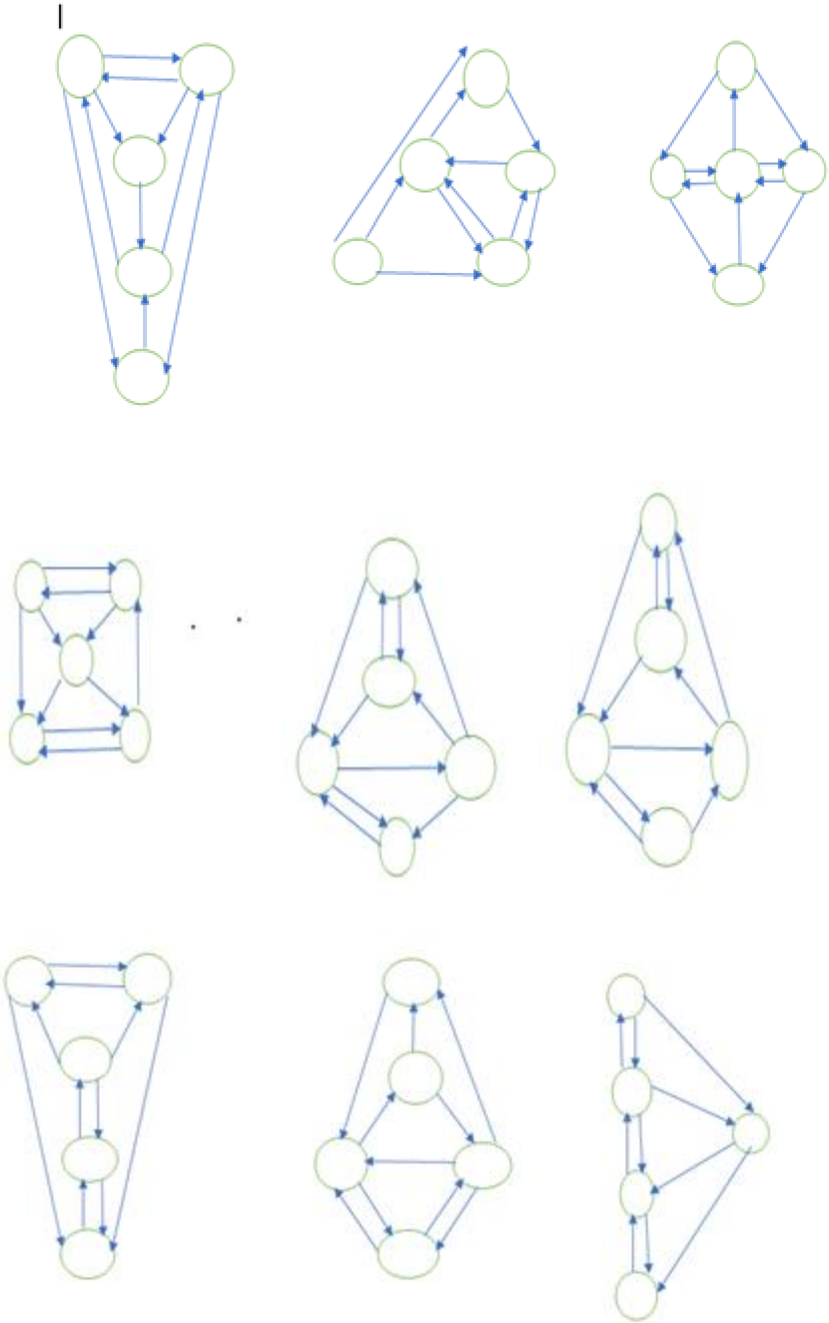
$$\lambda(x) + \sum_{(x,y)\in E(D)} \lambda(x,y) = m$$

For some $m \in Z$
In other words, an in-magic digraph is one where the sum of labels on edges directed in at each vertex is equal, but there is no restriction on the sums going out; and vice versa for an out magic digraph. Thus, any magic digraph is automatically both in-magic and out-magic. But not all graphs that are both in-magic and out-magic are magic.(2)
Given below is an example of In/Out magic graph.



VI.Distance Magic Labeling

A distance magic labeling or 1-vertex -magic labeling of a graph $G(V,E)$with $|V| = n$ is a mapping $\mu: V \to \{1,2, \dots n\}$ with the property that for every vertex $x \in V$ the weight of x,denoted by $w(x)$,which is equal to the sum of the labels of all neighbors of x,is always equal to a constant m,called the magic constant of μ.
As per observations Bloom et al used computer algorithms to examine the digraph with 5 vertices and 10 edges. There are nine digraphs which were found not to have a magic labeling. These nine directed graphs are shown below



So far there is no specific reason to why these nine digraphs cannot have magic labeling.(2)

III. Conclusion

In this paper we have done a survey on different types of magic labeling.
Further focus is to investigate on why the nine digraphs with five vertices and ten edges do not have a specific magic labeling.

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